ON THE CONCEPT OF LAW IN PHYSICS[1]

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Abstract

I discuss the main features of the concept of law in physics. Fundamental laws from Newtonian mechanics to general relativity are reviewed. I end with an outlook on the new form of laws in the emerging theory of quantum gravity.

[1]To appear in the Proceedings of the conference *The Concept of Law in Science*, Heidelberg, 4–5 June 2012.

1 Laws of Nature

The concept of law is widespread in both the sciences and the humanities. When one talks about laws of Nature, however, one usually refers to physics. What is a physical law? Richard Feynman, in his well known book *The Character of Physical Law* writes ([1], p. 13): "There is also a rhythm and a pattern between the phenomena of nature which is not apparent to the eye, but only to the eye of analysis; and it is these rhythms and pattern which we shall call Physical Laws." As a prototype of a physical law, Feynman states the law of gravitation.

As is evident from Feynman's quote, one needs a certain degree of abstraction from the phenomena to discern the laws of Nature. Without the eye of analysis, physical laws cannot be found.

In this essay, I shall briefly summarize the status of laws of Nature in modern physics and speculate about the development of new laws. A central role is there indeed played by gravitation. On the one hand, Einstein's theory of general relativity has introduced a dynamical spacetime into physics and has thus dramatically changed our attitude towards the formulation of fundamental laws. On the other hand, one expects that the consistent unification of general relativity with quantum theory will lead to a completely new type of law. For this reason, I shall discuss some aspects of fundamental laws as they appear in one approach to quantum gravity.

Physical laws are formulated within a physical theory or a set of theories. A theory consists of a set of mathematical equations and a set of mapping rules to phenomena in Nature. In the ideal case, these rules include a statement about their domain of validity.

In his famous book *Il Saggiatore*, Galileo Galilei has introduced the picture of Nature as a book written in mathematical language. This must, however, not be interpreted too literally. The mathematical language is not unique, and the same phenomena can be described in different ways. A good example is gravitation. In Newtonian terms, the motion of planets is described by differential equations containing an action at a distance. In Einsteinian terms, the planets move on geodesics in spacetime. If gravity is combined with quantum theory, yet another mathematical picture emerges. There is thus not a one-to-one relation between mathematics and reality. This is clearly expressed by a famous quote from Albert Einstein, who writes ([2], p. 119–120)

> Insofern sich die Sätze der Mathematik auf die Wirklichkeit beziehen, sind sie nicht sicher, und insofern sie sicher sind, beziehen sie sich

nicht auf die Wirklichkeit.[2]

According to Einstein, a certain degree of intuition is needed to find the correct laws of Nature; they cannot just be read off from the phenomena. Still, physical laws are not invented, but discovered, because they reflect properties of the real world, not just our imagination. In contrast to this, mathematical concepts are, in my opinion, invented. Why there are laws of Nature at all, is not obvious; nor is it a priori clear that we are able to discover them.

One can distinguish between physical laws at different levels. Here, we are mainly concerned with the fundamental laws, that is, laws that describe the fundamental interactions; examples are the laws of gravitation and electrodynamics. It is an open issue whether all these fundamental laws can be unified to one fundamental theory, often called ‘theory of everything’. If this happened, it would be the ultimate triumph of the reductionist programme in physics.

At a different level, one has effective laws such as the Second Law of thermodynamics. As we shall briefly discuss below, the Second Law seems to be a consequence of particular boundary conditions of our world, and it is open whether it can be derived in a different way from structures of a new theory, such as quantum gravity.

Yet another level concerns emergent laws for complex systems. They can, in principle, be derived from the fundamental laws, but show features that go much beyond those laws. In the words of Paul Anderson ([4], p. 395),

> . . . the whole becomes not only more than but very different from the sum of its parts.

Here, we shall not discuss such emergent laws, but focus on the fundamental physical laws from Newtonian mechanics to quantum gravity.

2 From Newtonian Mechanics to Special Relativity

A most important feature of our physical theories is the separation of the description into dynamical laws and initial conditions. This was expressed very clearly in Eugene Wigner’s Nobel speech ([3], p. 7–8),

[2] In so far the theorems of mathematics refer to reality, they are not certain, and in so far they are certain, they do not refer to reality.

> The regularities in the phenomena which physical science endeavors to uncover are called the laws of nature. ... The elements of the behavior which are not specified by the laws of nature are called initial conditions. ... The surprising discovery of Newton's age is just the clear separation of laws of nature on the one hand and initial conditions on the other. The former are precise beyond anything reasonable; we know virtually nothing about the latter.

Mathematically, our fundamental laws can be expressed as differential equations up to second order in space and time. They leave thus room for initial or (more generally) boundary conditions. Alternatively, the same laws can be expressed in integral form, as a variational principle, but this form is fully equivalent to the differential form.

Because the physical laws can be formulated as differential equations, they are completely deterministic. Determinism, in this modern sense, must not be confused with causality. If temporal boundary conditions are specified at a particular time, the solution of the equations is determined for any time, both before and after that particular time. Determinism is one of the most important concepts when discussing physical laws [5, 6].

Determinism does not yet mean predictability. Predictability presupposes determinism, but not the other way around [5]. Most systems in Nature exhibit chaotic behaviour, which means that small perturbations become exponentially large. Because this is not in conflict with determinism, one talks about 'deterministic chaos'. The prediction of the weather is a classic example, but already systems as simple as a double pendulum show chaotic behaviour. For the same reason, it is also not possible to predict the future of our Solar system, that is, the future of the motion of planets and asteroids, for more than about four million years.

Fundamental physical laws refer to dependences on space and time. It was one of Newton's great achievement to introduce the concepts of absolute space and absolute time to facilitate the formulation of his laws. To quote from his *Principia* ([7], p. 623),

> Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external. ... Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. ...

Let us consider Newton's second law of motion for the motion of a set of N particles described by their positions $\mathbf{x}_i$, $i = 1, \dots, N$,

$$m_i \frac{\mathrm{d}^2 \mathbf{x}_i}{\mathrm{d}t^2} = \mathbf{F}_i. \tag{1}$$

The force $\mathbf{F}_i$ on the ith particle is here assumed to be *given*. In the important case of gravitational interaction, it reads

$$\mathbf{F}_i = -G \sum_{j \neq i} \frac{m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|^2} \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|}. \tag{2}$$

Because (1) is a differential equation of second order in time, its solution is determined if position and velocity are specified at a particular moment of time.

The notions of absolute space and absolute time were criticized at several occasions in the history of science, mainly because these notions involve absolute (non-dynamical) elements. Among the critics were Berkeley, Leibniz, and Mach. Since, however, Newton's mechanics was extremely successful, attempts to formulate an alternative mechanics did not go very far [7]. Only after the advent of general relativity did people investigate models of classical mechanics without absolute space and time [8].

Besides gravitation, the only fundamental interaction that manifests itself at a macroscopic level is electrodynamics. It is described by the set of Maxwell's equations,

$$\begin{aligned} \nabla \mathbf{B} = 0 \quad &, \quad \nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \ , \\ \nabla \mathbf{E} = 4\pi\rho \quad &, \quad \nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}, \end{aligned} \tag{3}$$

where $\mathbf{B}$ and $\mathbf{E}$ are the magnetic and electric field, respectively. In contrast to Newton's equations, these are equations for local fields. Already Maxwell's contemporaries were impressed by the fact that these equations encode all the phenomena related to electricity, magnetism, and optics.[3] One of the main features of new fundamental laws is the fact that they can predict the occurrence of new phenomena. In the case of Maxwell's equations, these include the generation of radio waves, which proved to be of enormous technological significance.

In the formulation of physical laws, symmetry principles play a key role. Otherwise, it would almost be impossible to devise the correct equations out of the immense number of mathematical options. In classical mechanics, an important principle is the principle of relativity: the physical laws are invariant with respect to the transformation from one inertial frame into another. Maxwell's equations seem to violate this principle, because they contain a distinguished speed – the speed of light c. It was this apparent

[3]It was Boltzmann who cited Goethe's *Faust*: "War es ein Gott, der diese Zeichen schrieb?".

conflict between mechanics and electrodynamics that led Albert Einstein in 1905 to his special theory of relativity. By a careful analysis of the concept of time, he realized that Maxwell's equations do indeed obey the relativity principle, although the transformation law becomes more complicated (Lorentz instead of Galileo transformations). As Hermann Minkowski found out in 1908, special relativity can most clearly be formulated in terms of a four-dimensional union of space and time called spacetime (later simply called Minkowski space). Time by itself and space by itself are no longer absolute, but spacetime still is; its main characteristic is the presence of the lightcone structure, because the speed of light is the same in all inertial frames. It is then mandatory to formulate all physical laws in four-dimensional form so that covariance with respect to transformations between inertial systems is evident. Otherwise, it is not clear if new laws satisfy the relativity principle or not.

Quantum mechanics has introduced many new concepts into physics, but with respect to time nothing has changed; the theory has inherited Newton's absolute time. The time parameter t that occurs in the Schrödinger equation,

$$\hat{H}\Psi = \mathrm{i}\hbar\frac{\partial\Psi}{\partial t}, \tag{4}$$

is nothing but the time parameter of (1).

The Schrödinger equation (4) is a deterministic equation: if the quantum state Ψ is given at any particular instant of time, the solution follows for any other time value, both before and after that instant. The interpretation of Ψ is, however, drastically different from classical fields such as $\mathbf{E}$ or $\mathbf{B}$, because it is defined not in spacetime, but on a high-dimensional configuration space. Its connection with classical quantities is described by the probability interpretation. The emergence of classical behaviour is given by the process of decoherence [9].

If special relativity is combined with quantum theory, one arrives at quantum field theory. Here, four-dimensional flat Minkowski space is used as a rigid classical background on which the dynamics of the quantum fields is defined.

As many authors, in particular Albert Einstein, have noted, it is not natural to envisage something that can act but which cannot be acted upon (as is the case for Minkowski space). The situation changes dramatically with general relativity, to which we now turn.

3 General Relativity

In general relativity, the gravitational field is described by the geometry of a dynamical four-dimensional spacetime. The fundamental equations are a set of ten coupled partial differential equations for the metric $g_{\mu\nu}$. In standard notation, these Einstein field equations read

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{5}$$

For the first time, one is confronted with equations that are not formulated on a given spacetime, but equations that describe spacetime itself. One impressive example is the existence of gravitational waves, which describe the propagation of pure curvature without matter. As John Wheeler always emphasized, space tells matter how to move, and matter tells space how to curve.

In spite of the complex nature of the Einstein field equations, a well-defined initial value problem ('Cauchy problem') can be formulated. The metric coefficients $g_{\mu\nu}(x)$ can be determined uniquely (up to coordinate transformations) from appropriate initial data. An important feature in this context is the presence of four (at each space point) *constraints*. These constraints arise from the fact that the theory is invariant under four-dimensional diffeomorphisms ('coordinate transformations'). The initial data consist of the three-dimensional metric, the second fundamental form, and matter degrees of freedom on a spacelike hypersurface that satisfy the four constraints. In this way, spacetime itself is constructed from initial data. The existence of a well-defined Cauchy problem is of special relevance for numerical relativity, which is concerned with processes such as the evolution of two black holes orbiting each other.

General relativity is a very successful theory. With perhaps the exception of dark matter and dark energy, it describes all known gravitational phenomena. But it behaves also in an exemplary manner with respect to its limits. From general theorems ('singularity theorems'), one knows that there are situations in which the theory breaks down [10]. These are, in fact, important situations because they apply to the origin of our Universe and to the interior of black holes. For these and other reasons, one expects that the laws of gravity are, at the most fundamental level, not exactly described by Einstein's equations. One way to arrive at a more fundamental theory than general relativity is to take quantum theory into account. This will be the subject of the last section.

4 Quantum Gravity and Beyond

General relativity and quantum theory cannot both be exactly valid. One reason is what usually is referred to as the 'problem of time' [11]. Time is absolute in quantum mechanics (spacetime in quantum field theory), but it is dynamical in general relativity (as part of the dynamical spacetime). So what happens in situations where both theories become relevant?

If one keeps the linear structure of quantum theory and looks for a quantum wave equation that gives back Einstein's equations in the semiclassical limit, one arrives at a quantum constraint equation of the general form

$$\hat{H}\Psi = 0. \tag{6}$$

This equation is known as the Wheeler–DeWitt equation [11]. It has some amazing properties. The full quantum state Ψ of gravity and matter depends on the three-dimensional metric only, but is invariant under three-dimensional coordinate transformations. It does not contain any external time parameter t. The reason for this 'timeless' nature is obvious. In general relativity, a four-dimensional spacetime is the analogue to a particle trajectory in mechanics. After quantization, the trajectory vanishes, and so does spacetime. What remains is space, and the configuration space is the space of all three-geometries [12]. A constraint equation of the form (6) also occurs in loop quantum gravity [11].

To give a particular example, let us formulate the Wheeler–DeWitt equation for a simple cosmological model. For a closed Friedmann–Lemaître universe with scale factor a and a massive scalar field ϕ, this equation reads after an appropriate choice of units as follows (Λ is the cosmological constant):

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial\phi^2} - a + \frac{\Lambda a^3}{3} + m^2a^3\phi^2\right)\psi(a,\phi) = 0. \tag{7}$$

The timeless nature is evident. The cosmological wave function only depends on the two variable a and ϕ. As can be seen from the kinetic term, the Wheeler–DeWitt equation is of hyperbolic nature (this is also true for the general case). It does provides the means to define an *intrinsic time*, which is distinguished by the sign in the kinetic term. This intrinsic time, however, is no longer a time given from the outset, but is defined from the three-dimensional geometry itself. In this way, it resembles what the astronomers used to call ephemeris time [8].

The Wheeler–DeWitt equation thus represents a new type of physical law. It describes a timeless world at the most fundamental level. The usual time parameter of physics emerges only at an approximate level and under

very special circumstances [11]. To quote John Wheeler from his pioneering work ([13], p. 253),

> These considerations reveal that the concepts of spacetime and time itself are not primary but secondary ideas in the structure of physical theory. These concepts are valid in the classical approximation. However, they have neither meaning nor application under circumstances when quantum-geometrodynamical effects become important. ... There is no spacetime, there is no time, there is no before, there is no after. The question what happens "next" is without meaning.

In spite of its timeless nature, the Wheeler–DeWitt equation can, in principle, provide the means for an understanding of the arrow of time (the Second Law of thermodynamics mentioned above) [14]. Let us consider a Friedmann–Lemaître universe with scale factor $a \equiv \exp(\alpha)$ and small perturbations symbolically denoted by x_n. The Wheeler–DeWitt equation then assumes the form

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial\alpha^2} + \sum_n \left[-\frac{\partial^2}{\partial x_n^2} + \underbrace{V_n(\alpha, x_n)}_{\to 0 \text{ for } \alpha\to-\infty} \right]\right)\Psi = 0. \tag{8}$$

The potentials V_n have the property that they vanish when the scale factor goes to zero (i.e., near the big bang and the big crunch). This expresses a fundamental asymmetry of the Wheeler–DeWitt equation. For small scale factor, therefore, one can have solutions that are fully unentangled among the degrees of freedom. But for increasing a the solution becomes entangled, and one can obtain a non-vanishing entanglement entropy upon tracing out the perturbations. Entanglement entropy can be related to thermodynamic entropy, and this entropy then increases with increasing size of the universe and thereby defines a definite direction. In the limit where an approximate time parameter is present, this gives rise to the usual Second Law. But if viewed from this fundamental perspective, the expansion of the universe is a pure tautology.

So what is the future of physical laws? The Wheeler–DeWitt equation has not yet been experimentally tested, but it is an equation that follows in a straightforward way from the unification of quantum theory with gravity. It describes quantum effects of gravitation, but does not encompass by itself a unification of all interactions. A candidate for a unified theory is string theory. The structure of the fundamental laws in this approach is not yet

fully understood, but it seems to be different from the structures discussed above [11].

Whether a fundamental 'theory of everything' can be found, is open. It may happen that such a theory will be available in this century and that the fundamental picture in physics is 'complete' in the sense that all phenomena can be derived from it, at least in principle. Or it may happen that we are stuck because experimental progress becomes slower and slower and no decision among candidates for a fundamental theory can be made. In one way or another, it is true what Feynman wrote already in 1964 ([1], p. 172): "The age in which we live is the age in which we are discovering the fundamental laws of nature, and that day will never come again. It is very exciting, it is marvellous, but this excitement will have to go."

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